Deflection of Flat-Plate Slabs

A comparative study of the effects of high-strength concrete and high-strength reinforcement

by Cathy I-Chi Huang, David E. Hoy, Yun Jennifer Lan, Camille de Romémont, and Ramon E. Gilsanz

High-strength steel reinforcement (HSR) has been accepted as an effective alternative for reinforcing structural walls, columns, and beams. However, less attention has been given to the use of HSR for reinforcing flat-plate concrete slabs.[1]

The structural design and dimensioning of reinforced concrete slabs are often governed by deflection limits, punching shear resistance, and constraints imposed by mechanical, electrical, and plumbing (MEP) or architectural design requirements. The use of reinforcement with 80 or 100 ksi (550 to 690 MPa) yield strength enables a reduction in the amount of steel required. While this reduction can lower the construction cost, it may also result in greater slab deflections. However, the potential for increased deflection can be counteracted using high-strength concrete, as this will increase the modulus of elasticity and modulus of rupture.

Furthermore, the cost savings that may be achieved by using high-strength reinforcement outweigh the added cost of high-strength concrete. According to RSMeans,[2] an 8 in. (200 mm) slab reinforced with Grade 80 No. 4 bars is $0.42/ft^2 less expensive than the same slab reinforced with Grade 60 No. 4 bars, while increasing the concrete strength of the slab from 5000 to 8000 psi (35 to 55 MPa) increases material costs by only $0.26/ft^2.

These competing factors create opportunities for optimizing the cost and performance of reinforced concrete flat-plate slabs. Using a slab prototype from an existing building, this study compares designs using different strengths of reinforcing bars and concrete.

Concrete Slab

Geometric definition and applied loads

Our prototype is a typical floor slab for a 21-story residential tower in New York, NY (Fig. 1). The floor plate is 50 x 70 ft (15.2 x 21.3 m) in plan, and a balcony cantilevers from the north façade. The vertical force-resisting elements consist of columns along the façade and 12 in. (305 mm) thick shear walls at the stair and elevator openings. The column dimensions are 10 x 48 in. (255 x 1220 mm) and 10 x 72 in. (255 x 1830 mm) for rectangular columns, 28 in. (710 mm) diameter for circular columns, and 36 x 48 x 10 in. (915 x 1220 x 255 mm) for L-shaped columns. The slab is 8 in. thick

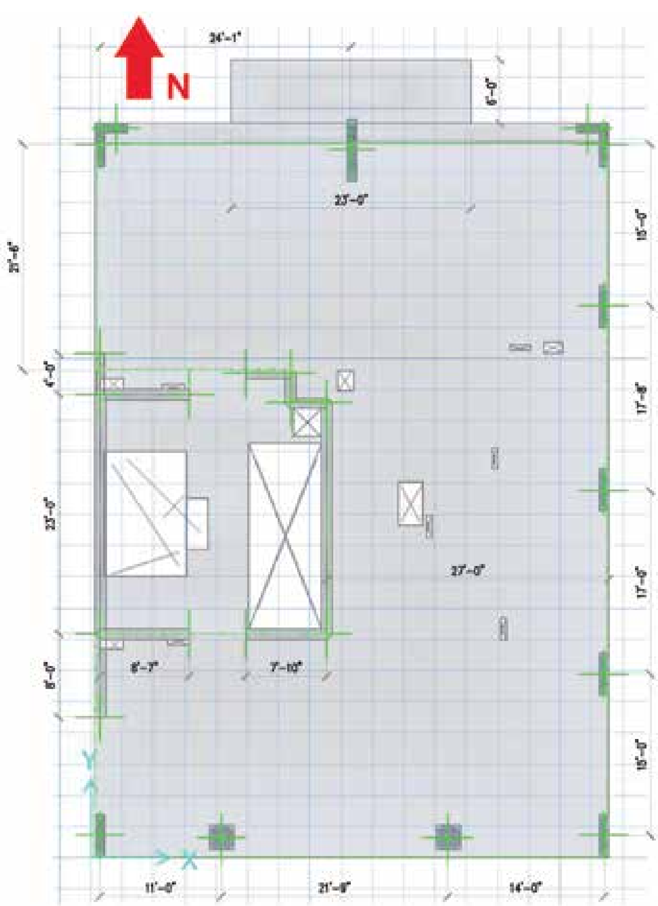

Fig. 1: Floor plan of the studied slab (Note: ' = ft and " = in.; for example, 4'-0" = 4 ft 0 in.)

outside the shear wall core and 18 in. (460 mm) thick inside the core.

The applied loads for the slab design are governed by the New York City Building Code (NYCBC). The superimposed dead load (SUPERDEAD) is 18 psf (0.86 kPa) for partitions, finishes, and mechanical equipment, except on the balcony where SUPERDEAD is 3 psf (0.14 kPa) to account for waterproofing. The floor live load is 40 psf (1.9 kPa) at the interior and 60 psf (2.9 kPa) at the balcony. A 250 pound per linear foot (plf) (3650 N/m) line load is applied on the north and south façades to account for a glass curtainwall façade weighing 25 psf (1.2 kPa). A 470 plf (6900 N/m) line load is applied on the east and west façades to account for a 6 in. (150 mm) concrete masonry unit (CMU) wall and finishes, weighing 48 psf (2.3 kPa). All mechanical, stair, and elevator openings in the building have been considered in the analyses. Lateral loads are not considered in this study.

Material properties

We evaluated designs using normalweight concrete with compressive strengths f_c' of 5000, 8000, and 10,000 psi (35, 55, and 70 MPa) for the slab and 8000 psi for columns and walls. The concrete modulus of elasticity E_c was computed per Section 8.5.1 of ACI 318-11[3]: $E_c = 57{,}000\sqrt{f_c'}$. The modulus of rupture f_r for the slab was taken as $4\sqrt{f_c'}$ psi ($0.33\sqrt{f_c'}$ MPa), in accordance with recommendations for two-way concrete slabs in Section 4.3.3 of ACI 435R-95.[4] We chose this as conservative relative to the $7.5\sqrt{f_c'}$ psi ($0.62\sqrt{f_c'}$ MPa) value for normalweight concrete, as specified in Section 9.5.2.3 of ACI 318-11, and the $5\sqrt{f_c'}$ value when $f_y \geq 80$ ksi ($0.41\sqrt{f_c'}$ MPa when $f_y \geq 550$ MPa), as specified in Section 8.3.1.1 of ACI 318-19.[5] We did not study the effects of the new limiting value on net tensile strain defining tension-controlled sections (refer to ACI 318-19, Table 21.2.2). We further acknowledge that designers should base their analyses on test data obtained using local materials, as E_c and f_r may vary widely from the values predicted using equations in industry standards and reports. In summary, E_c values were 4000, 5100, and 5700 ksi (28, 35, and 39 GPa) for f_c' values of 5000, 8000, and 10,000 psi, respectively; and f_r values were 280, 360, and 400 psi (1.9, 2.5, and 2.8 MPa) for f_c' values of 5000, 8000, and 10,000 psi, respectively.

For calculation of long-term deflections, we used creep and shrinkage coefficients of 1.9458 and 0.000579, respectively, based on recommendations per ACI 209.2R-92, Appendix A,[6] for an 8 in. slab. We did not decrease creep coefficients with increased concrete strength, which would have reduced deflections further.

We analyzed the prototype slab with either Grade 60 or Grade 80 reinforcing bars. Although splice length varies inversely with $\sqrt{f_c'}$ and directly with f_y, we used a single multiplier of 1.3 to account for splices and anchorage of bars.

To limit crack widths due to temperature and shrinkage, Section 7.12.2.1of ACI 318-11 requires a minimum reinforcement ratio, based on the gross slab area and in two orthogonal directions, of 0.0018 for Grade 60 bars and 0.0014 for Grade 80 bars. In this study, this minimum is provided by the bottom reinforcement each way, as required per Section 1917.2.1.1 of the 2008 NYCBC.[7] It should be noted that ACI 318-19, Section 24.4.3.2, now requires a minimum reinforcement ratio of 0.0018, regardless of the bar grade, so the benefits of using high-strength bars will be reduced relative to our analysis.

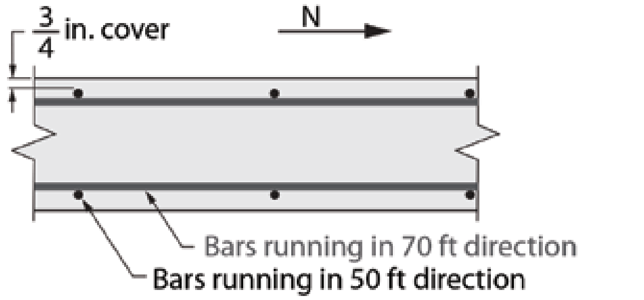

Fig. 2: Section of the 8 in. (200 mm) slab (Note: 70 ft = 21.3 m; 3/4 in. = 19 mm)

Reinforcement layout

The NYCBC requires a typical two-way bottom reinforcing mat in the slab. In New York City, opinions vary among engineers regarding the need to also provide a continuous two-way top reinforcing mat. Although a bar layout with a continuous top mat requires more steel than the alternative, it can be placed faster because it is more repetitive and requires fewer additional top bars in negative moment regions. Further, some builders contend that mats with No. 5 bars are preferred over mats with No. 4 bars because they are easier to walk on, are less susceptible to deformation during placement, and further reduce the number of additional bars in negative moment regions.

In these analyses, typical top mats consisting of either No. 4 or No. 5 bars are compared. As illustrated in Fig. 2, the outer layer of reinforcing is placed in the direction of the shorter span, with a clear cover of 3/4 in. (19 mm).

Description of finite element analysis

Our analyses were performed using CSI SAFE[8] finite element software. The columns and walls were modeled for one story above and one story below the slab. Except for vertical translation of the upper column and wall elements, all degrees of freedom were constrained at the extreme ends.

The total long-term slab deflections were obtained through a nonlinear cracked analysis, considering both immediate and long-term effects of cracking. Short-term analyses used the cracked section moment of inertia, based on the amount of specified reinforcing, to calculate deflections wherever the slab stresses exceeded the modulus of rupture. The long-term cracked analyses used the cracked moment of inertia and accounted for creep and shrinkage by adjusting the modulus of elasticity with the coefficients mentioned previously. The long-term analyses included only sustained loads, the slab

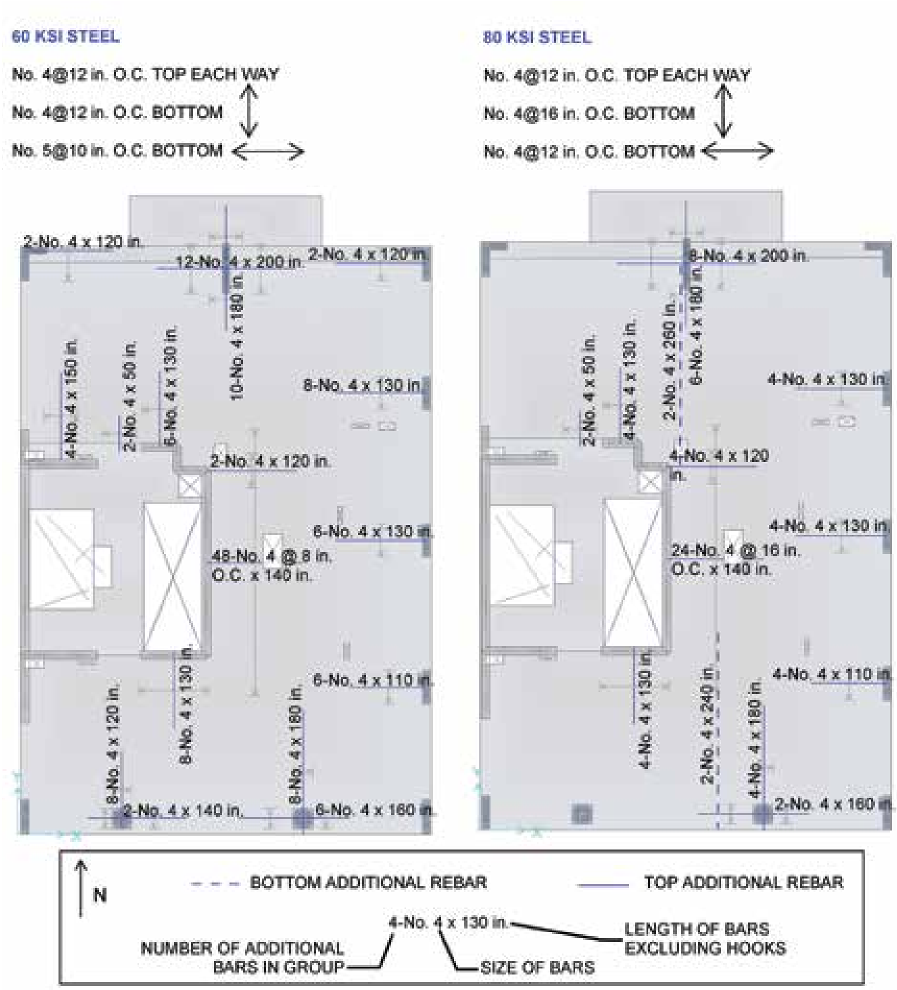

Fig. 3: Reinforcing steel for the prototype slab with Grade 60 and Grade 80 bars. Top mats comprise No. 4 bars at 12 in. on center each way. The typical top and bottom mats are indicated above each floor plan; and additional top or bottom reinforcing bars required for strength are shown on each floor plan (Note: 1 in. = 25 mm)

self-weight, and superimposed dead loads, while the short-term analyses included both sustained and transient loads.

Three cases were analyzed and combined to obtain the final deflections:

- Case 1: Long-term deflection under DEAD + SUPERDEAD;
- Case 2: Short-term deflection under DEAD + SUPERDEAD + LIVE; and
- Case 3: Short-term deflection under DEAD + SUPERDEAD.

The total deflection was calculated using the combination of Case 1 + (Case 2 – Case 3).

Reinforcing steel for the slab was calculated by the SAFE program based on an elastic finite element analysis. The typical bottom mat was specified to meet temperature and shrinkage requirements and structural integrity requirements and to minimize the amount of additional bottom bars required.

Results

The amounts of reinforcement required in the slab are shown in Fig. 3 and 4. Inside the core, the minimum reinforcement required for shrinkage and thermal strain (No. 5 at 9 in. [229 mm] on center for 60 ksi bars and No. 5 at 12 in. on center for 80 ksi bars) was applied as the typical bottom mat in both directions. Outside the core, we used a typical top mat of No. 4 at 12 in. in each direction (Fig. 3) or No. 5 at 12 in. in each direction (Fig. 4). In accordance with the 2008 NYCBC 1917.2.2, continuous peripheral tie reinforcements consisting of two bars of the same size as the typical mat were placed at the top and bottom of the slab perimeter. Although there are link beams connecting the shear walls, the link beam reinforcement is not shown.

Compared to the design based on Grade 60 bars, the design based on Grade 80 bars requires less steel—there are fewer additional bars and the typical mat bars have greater spacing. Table 1 shows the weight of reinforcing steel relative to slab area for the two top mat designs. These weights include all typical mat, edge bars, and additional bars, as well as 30% additional length for hooks and splices. Using Grade 80 bars reduces the necessary amount of steel by 20 to 25%.

We used the same bar layouts in computations of the total slab deflection for the different concrete strengths. Figure 5 shows the long-term deformed shape of the slab under the applied dead and live loads described previously. Table 2 shows comparisons of deflections calculated for slabs with 5000, 8000, and 10,000 psi concrete.

The results are reported with an accuracy of two significant figures to illustrate the magnitude of difference. Practically, however, differences in deflection under 1/8 in. (3 mm) are considered negligible in the field.

While the use of Grade 80 bars rather than Grade 60 bars results in increases in deflection of 5 to 15%, the analyses show that the higher E_c and f_r gained using high-strength concrete can reduce overall deflections. To better understand the effects of reinforcing steel and concrete strength, deflections are calculated for the No. 4 at 12 in. typical top bar layout for concrete compressive strengths ranging from 5000 to 10,000 psi. Figure 6 shows the calculated deflections as functions of steel and concrete strengths.

In addition to analyzing the slabs for f_c' values of 5000, 8000, and 10,000 psi, we calculated deflections for slabs with tensile stresses near f_r. The curves in Fig. 6 exhibit abrupt drops in deflections for f_c' of about 6700 psi for the slab with Grade 60 bars and about 7300 psi for the slab with Grade 80 bars. At concrete strengths above these values, the slabs remain uncracked.

Slabs with Grade 60 bars exhibit this drop at a lower concrete strength than slabs with Grade 80 bars because the gross moment of inertia of the slab with Grade 60 bars is higher than the gross moment of inertia of the slab with Grade 80 bars. In other words, the tensile stress in the concrete is lowered by higher steel content. Because the quantity of steel has a dominant effect on the moment of inertia for a cracked section, the difference between the two curves is larger for concrete with f_c' below 7000 psi.

Conclusions

We analyzed the effects of reinforcing steel and concrete strengths on slab deflection. The results demonstrate that the deflection resulting from the use of a lower amount of high-strength reinforcing steel can be counteracted by using high-strength concrete. Prior to concrete cracking, the slab deflection depends less on the reinforcement ratio.

The results also indicate that using high-strength reinforcing steel and high-strength concrete in slabs can reduce cost and improve serviceability. With a conventional design including 5000 psi concrete, Grade 60 bars, and a typical top mat of No. 4 bars at 12 in., the prototype flat-plate slab contains 5.1 psf (25 kg/m^2) of steel and deflects 0.91 in. (23 mm). In contrast, when 10,000 psi concrete and Grade 80 bars are used, the prototype slab requires only 3.9 psf (19 kg/m^2) of steel (about a 24% reduction) and the deflection is only 0.58 in. (15 mm) (about a 36% reduction). Final savings will be determined by the general contractor, concrete supplier, and the trades that agree to use less steel and higher-strength concrete.

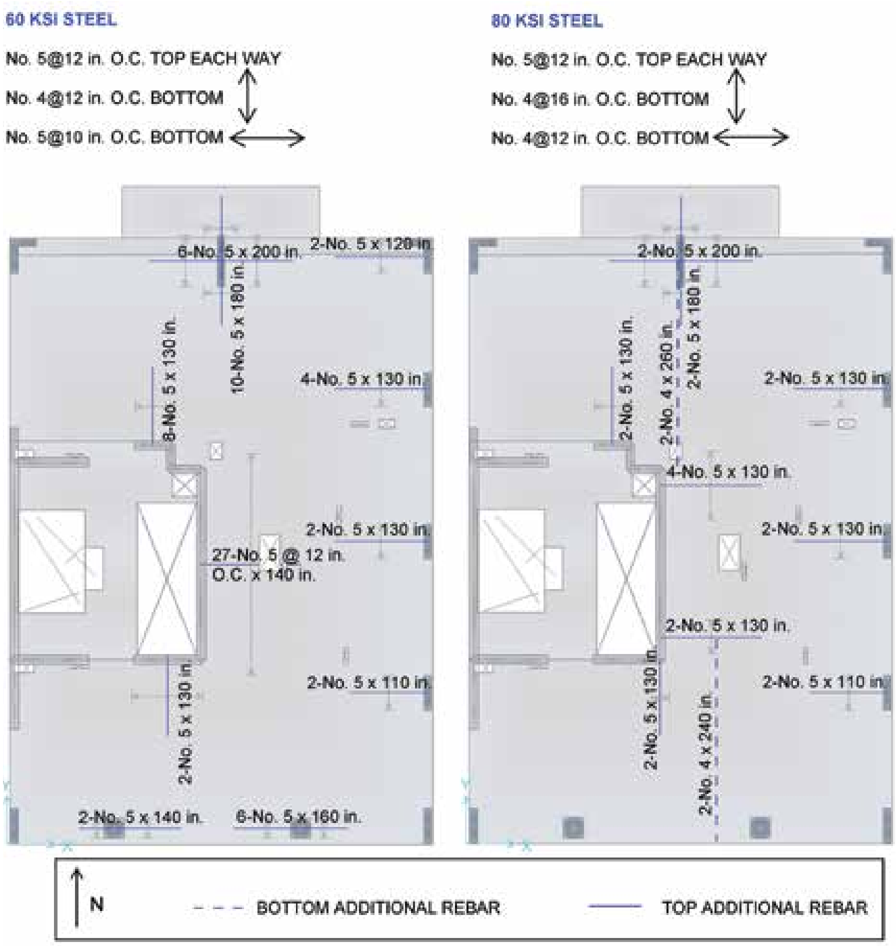

Fig. 4: Reinforcing steel for the prototype slab with Grade 60 and Grade 80 bars. Top mats comprise No. 5 bars at 12 in. on center each way. The typical top and bottom mats are indicated above each floor plan; and additional top or bottom reinforcing bars required for strength are shown on each floor plan (Note: 1 in. = 25 mm)

Table 1:
Reinforcing steel weight relative to slab area

Top mat design	Reinforcing steel weight, psf — Grade 60 bars	Reinforcing steel weight, psf — Grade 80 bars
No. 4 at 12 in.	5.1	3.9
No. 5 at 12 in.	6.0	4.8

Note: 1 psf = 4.9 kg/m^2; 1 in. = 25 mm

Table 2:
Maximum calculated deflection as a function of concrete strength and bar grade for typical top bar mats

Steel type	Deflections for various concrete strengths, in.					
	5000 psi		8000 psi		10,000 psi	
	No. 4 bars at 12 in.	No. 5 bars at 12 in.	No. 4 bars at 12 in.	No. 5 bars at 12 in.	No. 4 bars at 12 in.	No. 5 bars at 12 in.
Grade 60	0.91	0.87	0.66	0.64	0.51	0.50
Grade 80	1.00	0.96	0.68	0.70	0.58	0.58

Note: 1 psi = 0.007 MPa; 1 in. = 25 mm

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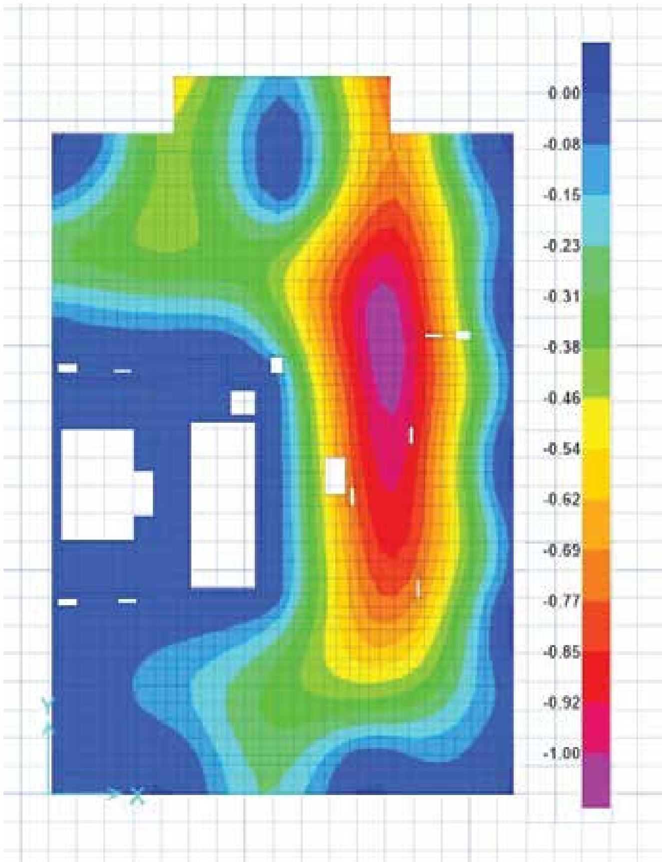

Fig. 5: Vertical deflection due to gravity loading with 80 ksi steel and 5000 psi concrete (scale is in in.) (Note: 1 in. = 25 mm)

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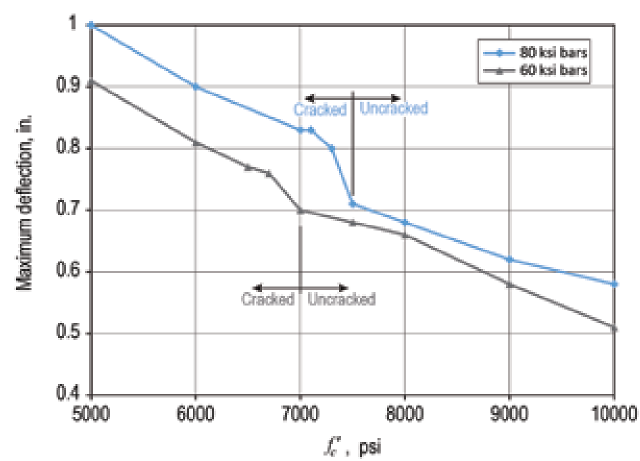

Fig. 6: Comparison of deflections calculated for slabs with Grade 80 and Grade 60 bars. Calculations were made for the slab with No. 4 at 12 in. typical top bar layout (Note: 1 in. = 25 mm, 1 psi = 0.007 MPa)

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Selected for reader interest by the editors.

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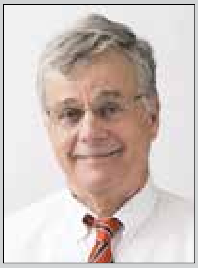

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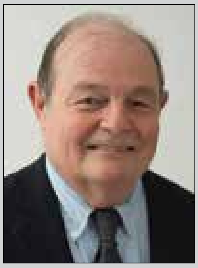

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